Summer Work for AP Calculus AB 2018-2019

I am looking forward to working with you next year in Calculus. This letter is attached to a packet. The packet is due on the first day of school. Try to find the correct answers using algebraic methods—not calculators. If you cannot get an answer at first, don't stress out, just try your best and show your work. If all else fails, get out your calculator to complete the problem. Feel free to discuss with friends. However, just getting the answers from someone will not be beneficial. These questions and topics are specifically chosen as review for the upcoming course.

Next year AP Calculus students will be using the TI-89 calculators. We have a large supply of TI-89s to rent out to students for $10 for use during the school year. However, if you want to buy your own, you may be able to find a good used one on eBay. I would suggest buying a TI-89 OR the TInspire CAS. The Inspire is a more updated calculator.

Timeline for the **first test** in Calculus

Summer: You will find and print this packet. You will work to complete this packet, without a calculator. Do not wait until August 5th to start this work.

August 6th (day 1): You will take the first portion of the test; this will be over the Unit Circle in which you will be required to have the Unit Circle memorized and be able to answer trigonometric value questions based on the Unit Circle. This will be worth **25** points.

August 6th (day 1): You will be required to turn in the completed packet. This will be graded for accuracy and completion. This will be worth **25** points. You will lose 5 points per day for every day it is late.

August 7th – August 9th (day 2, day 3, day 4): I will answer questions about the summer work during class and before or after school as needed.

August 10th (day 5): We will take a test over the summer review material. The test will consist of calculator and no calculator questions. This will be worth **50** pts.

Together all these parts make for the first **100** point test.

See you in the fall. Have a great summer. If you have any questions, feel free to contact me by email:

sweatman.melinda@mail.fcboe.org

Sample items for Unit circle test (all radians):

1. sin $\frac{\pi}{6}$ 2. Cos $\frac{3\pi}{4}$ 3. Tan $\frac{\pi}{3}$ 4. Csc $\frac{7\pi}{6}$ 5. sin $\frac{\pi}{3}$

6. cos 2π 7. Tan $\frac{5\pi}{3}$ 8. Sec $\frac{5\pi}{4}$ 9. Sin $\frac{7\pi}{4}$ 10. cot π

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

6. $f(2) =$ ____________ 7. $g(-3) =$ ______________ 8. $f(t+1) =$ ___________

9. $f\left[g(-2)\right] =$ __________ 10. $g\left[f(m+2)\right] =$ ___________ 11. $\dfrac{f(x+h) - f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ __________ 13. $f\left(\dfrac{2\pi}{3}\right) =$ ____________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

14. $h\left[f(-2)\right] =$ _______ 15. $f\left[g(x-1)\right] =$ _______ 16. $g\left[h(x^3)\right] =$ _______

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

42.5 Find the equation of a line passing though the point (2, 8) and perpendicular to the line 5x - 6y = 6

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

35. Find the value of $\log_8 32$

36. Find $\log 8 + \log 50 - \log 4$

37. Find $\frac{5}{2}\ln\frac{1}{e^{2/5}}$

38. Solve for x: $\log_9\left(x^2 - x - 3\right) = \frac{1}{2}$

39. Solve for x: $\log_5(15x + 5) - \log_5 x = 2$

40. Solve for x: $4^x = 5^{x-1}$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

89. $f(x) = \frac{1}{x^2}$

90. $f(x) = \frac{x^2}{x^2 - 4}$

91. $f(x) = \frac{2 + x}{x^2(1 - x)}$

Determine all Horizontal Asymptotes.

92. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

93. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

94. $f(x) = \frac{4x^5}{x^2 - 7}$

50. $a.)\ \sin 180^\circ$

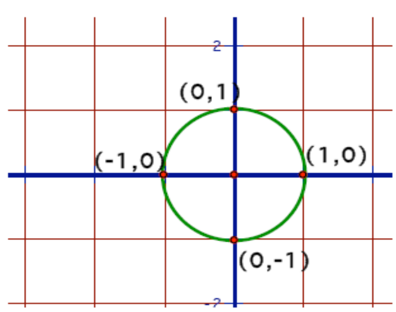
$b.)\ \cos 270^\circ$

$c.)\ \sin(-90^\circ)$

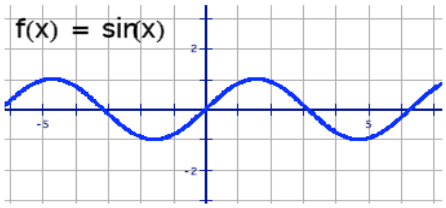
$d.)\ \sin \pi$

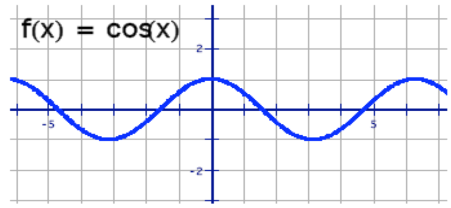
$e.)\ \cos 360^\circ$

$f.)\ \cos(-\pi)$



Graphing Trig Functions





y = sin x and y = cos x have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

51. $f(x) = 5\sin x$

52. $f(x) = \sin 2x$

53. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

54. $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

57. $\cos 2x = \frac{1}{\sqrt{2}}$

58. $\sin^2 x = \frac{1}{2}$

59. $\sin 2x = -\frac{\sqrt{3}}{2}$

60. $2\cos^2 x - 1 - \cos x = 0$

61. $4\cos^2 x - 3 = 0$

62. $\sin^2 x + \cos 2x - \cos x = 0$

Guidelines for verifying trigonometric identities:

1) Your job is to prove one side of an identity is equal to the other so you will only work on one side of the identity, so…
2) Always work on the most complicated side and try to transform it to the simpler side. More complicated can mean the side that is "longer" or has more complicated expressions. Additions (or subtractions) are generally more complicated than multiplications.
3) If an expression can be multiplied out, do so.
4) If an expression can be factored, do so.
5) If you have a polynomial over a single term, you can "split it" into several fractions.
6) If you have an expression, that involves adding fractions, do so finding a lowest common denominator.
7) When in doubt, convert everything to sines and cosines.
8) Don't be afraid to create complex fractions. Once you do that, many problems are a step away from solution.
9) Always try something! You don't have to see the solution before you actually do the problem. Sometimes when you try something, the solution just evolves.

3) $\sin x\left(\csc x+\sin x\sec^2 x\right)=\sec^2 x$

4) $2\cos^2 x+\sin^2 x=\cos^2 x+1$

5) $2\cos^2 x-1=1-2\sin^2 x$

6) $\left(\sin x+\cos x\right)^2+\left(\sin x-\cos x\right)^2=2$

7) $\dfrac{\cot x}{\csc x}=\cos x$

8) $\tan x+\cot x=\sec x\csc x$

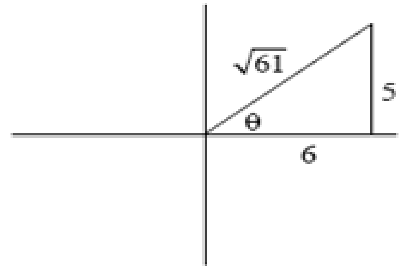
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta=\frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

63. $\tan\left(\arccos\frac{2}{3}\right)$

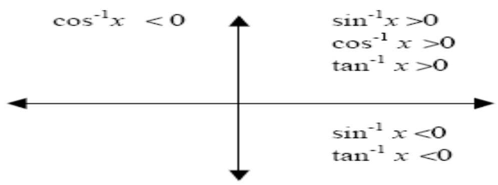
64. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$\arcsin(x)$ $\sin^{-1}(x)$

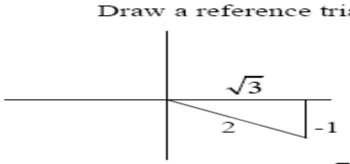
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:

Express the value of "y" in radians.

$y=\arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y=-\frac{\pi}{6}$ so that it falls in the interval from

$\frac{-\pi}{2}<y<\frac{\pi}{2}$ Answer: $y=-\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

76. $y=\arcsin\dfrac{-\sqrt{3}}{2}$

77. $y=\arccos(-1)$

78. $y=\arctan(-1)$